SOLUTION OF TRIANGLES

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 C

Given A : B : C = 3 : 5 : 4

$\because$ A + B + C=180º

$\Rightarrow$ A =45º, B = 75º, C = 60º

$$\Rightarrow \frac{a}{\sin 45^\circ} = \frac{b}{\sin 75^\circ} = \frac{c}{\sin 60^\circ}$$

$$\Rightarrow \frac{\sqrt{2}a}{1} = \frac{2\sqrt{2}b}{\sqrt{3}+1} = \frac{2c}{\sqrt{3}}$$

$$\Rightarrow a = \frac{2b}{\sqrt{3}+1} = \frac{\sqrt{2}c}{\sqrt{3}}$$

by I & III $\Rightarrow$ $\sqrt{3}$ a = $\sqrt{2}$ c ....(i)

by I & II $\Rightarrow$ ($\sqrt{3}$ + 1) a = 2b ....(ii)

$\therefore$ a + b + $\sqrt{2}$ c = a + b + $\sqrt{3}$ a {by (i)}

= b + ($\sqrt{3}$ + 1) a = b + 2b = 3b {by (ii)}

Sol.2 C

If $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{\cos A}{\sin A} = \frac{\cos B}{\sin B} = \frac{\cos C}{\sin C}$$

$\Rightarrow$ cotA = cotB = cotC $\Rightarrow$ A = B = C

$\therefore$ Δ is equilateral

Sol.3 C

$\because$ A + B + C = π $\Rightarrow$ A = π – (B + C)

Now $\frac{bc\sin^2 A}{\cos A + \cos B\cos C}$

$$= \frac{bc\sin^2 A}{\cos(\pi-(B+C)) + \cos B\cos C}$$

$$= \frac{bc\sin^2 A}{-\cos(B+C) + \cos B\cos C}$$

$$= \frac{bc\sin^2 A}{-\cos B\cos C + \sin B\sin C + \cos B\cos C}$$

$$= \frac{bc\sin^2 A}{\sin B\sin C} = \frac{bck^2a^2}{k^2bc} = a^2$$

$$\left\{\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{let})\right\}$$

Sol.4 C

(a + b + c) (b + c – a) = k bc

$\Rightarrow$ (b + c + a) (b + c – a) = kbc

$\Rightarrow$ $(b + c)^2 - a^2$ = kbc

$\Rightarrow$ $b^2 + c^2 - a^2 + 2bc$= kbc

$\Rightarrow$ $b^2 + c^2 - a^2$ = (k – 2) bc

$$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{k-2}{2}$$

$$\because -1 < \frac{k-2}{2} < 1$$

$\Rightarrow$ –2 < k – 2 < 2 $\Rightarrow$ 0 < k < 4

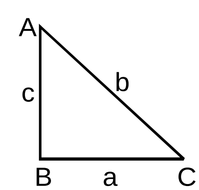
Sol.5 B

Given s – a = 3 $\Rightarrow \frac{a+b+c}{2} - a = 3$

–a + b + c = 6 ....(1)

& s – c = 2 $\Rightarrow \frac{a+b+c}{2} - c = 2$

a + b – c = 4 ....(2)



& s = a + 3, s = c + 2

$\Rightarrow$ a + 3 = c + 2 $\Rightarrow$ c – a = 1

from (1) b + c – a = 6 $\Rightarrow$ b + 1 = 6 $\Rightarrow$ b =5

$b^2 = a^2 + c^2$ $\Rightarrow$ $b^2 = a^2 + (a + 1)^2$

$\Rightarrow$ $25 = 2a^2 + 2a + 1$ $\Rightarrow$ $2a^2 + 2a - 24 = 0$

$\Rightarrow$ $a^2 + a - 12 = 0$ $\Rightarrow$ (a + 4) (a – 3) = 0

$\Rightarrow$ a = 3 ($\therefore$a = –4 not possible)

$\therefore$ c = a + 1 $\Rightarrow$ c = 4

Sol.6 C

$$\Rightarrow 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 3\sin(B+C)$$

$$\Rightarrow \cos\left(\frac{B-C}{2}\right) = 3\cos\left(\frac{B+C}{2}\right)$$

$$\Rightarrow \cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 3\cos\frac{B}{2}\cos\frac{C}{2} - 3\sin\frac{B}{2}\sin\frac{C}{2}$$

$$\Rightarrow 4\sin\frac{B}{2}\sin\frac{C}{2} = 2\cos\frac{B}{2}\cos\frac{C}{2}$$

$$\Rightarrow \cot\frac{B}{2}\cot\frac{C}{2} = 2$$

Sol.7 B

Given $A=\frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ cm

& area of $\Delta ABC = \frac{9\sqrt{3}}{2}$ cm^2

Now $\Delta = \frac{1}{2}bc\sin A$

$$\Rightarrow \frac{9\sqrt{3}}{2} = \frac{1}{2}bc\frac{\sqrt{3}}{2} \Rightarrow bc = 18$$

$\because a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow a^2 = (b-c)^2 + 2bc - 2bc\cos A$

$$\Rightarrow a^2 = 27 + 36\left(1+\frac{1}{2}\right)$$

$\Rightarrow a^2 = 27 + 54 = 81 \Rightarrow a = 9$

Sol.8 B

Given In a ΔABC, $\sin A = \frac{2\Delta}{bc}$

$$\Rightarrow \sin A = \frac{2[a^2-b^2-c^2+2bc]}{bc}$$

$\Rightarrow \sin A = 2[2 - 2\cos A]$

$\Rightarrow \sin A = 4[1 - \cos A]$

$$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2} = 4 . 2\sin^2\frac{A}{2}$$

$$\Rightarrow \tan\frac{A}{2} = \frac{1}{4}$$

$$\Rightarrow \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \frac{2\times\frac{1}{4}}{1-\left(\frac{1}{4}\right)^2}$$

$$= \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{1}{2}\times\frac{16}{15} \Rightarrow \tan A = \frac{8}{15}$$

Sol.9 B

$$\frac{b^2-c^2}{2aR} = \frac{k^2\sin^2 B - k^2\sin^2 C}{a\left(\frac{a}{\sin A}\right)}$$

$$= \frac{k^2\sin^2 B - k^2\sin^2 C}{\frac{k^2\sin^2 A}{\sin A}} = \frac{\sin^2 B - \sin^2 C}{\sin A}$$

$$= \frac{\sin(B+C)\sin(B-C)}{\sin(B+C)} \quad \{\because A = \pi - (B+C)\}$$

$= \sin(B - C)$

Sol.10 B

Given $b = 2$, $c = \sqrt{3}$ $\angle A = \frac{\pi}{6}$

$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

$$= 4 + 3 - 4\sqrt{3}\frac{\sqrt{3}}{2} = 7 - 6 = 1$$

$$\therefore \frac{a}{\sin A} = 2R \Rightarrow R = \frac{a}{2\sin A} = \frac{1}{2\times\frac{1}{2}} = 1$$

Sol.11 A

$$\frac{a\cos A + b\cos B + c\cos C}{a+b+c} \quad \{a = 2R\sin A\}$$

$$= \frac{2R(\sin A\cos A + \sin B\cos B + \sin C\cos C)}{2R(\sin A + \sin B + \sin C)}$$

$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2(\sin A + \sin B + \sin C)}$$

$$= \frac{4\sin A\sin B\sin C}{8\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} \text{ (By using Identity)}$$

$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{r}{R}$$

$$\{\therefore r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\}$$

Sol.12 B

Given a : b : c = 3 : 7 : 8

We know $R = \frac{a}{2\sin A}$ & $r = \frac{\Delta}{s}$

$$\Rightarrow \frac{R}{r} = \frac{as}{2\Delta \sin A} = \frac{a(a+b+c)}{4\times\frac{1}{2}bc\sin^2 A}$$

$$= \frac{a(a+b+c)}{2bc\sin^2 A} \quad \{\Delta = \frac{1}{2} \text{ bc sin A}\}$$

$\{\because$ a = 3k, b= 7k, c = 8k$\}$

$$\Rightarrow \frac{R}{r} = \frac{3(3+7+8)}{2\times7\times8\sin^2 A}\frac{k^2}{k^2} = \frac{3\times18\times14\times14}{2\times7\times8\times27}$$

$$\Rightarrow \frac{R}{r} = \frac{14}{4} = \frac{7}{2} \Rightarrow \text{ R : r = 7 : 2}$$

$$\because \cos A = \frac{b^2+c^2-a^2}{2bc}$$

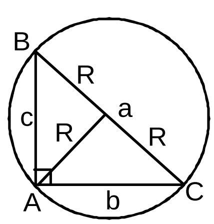
$$= \frac{k^2}{k^2}\ \frac{49+64-9}{2.7.8} = \frac{104}{2.7.8} = \frac{13}{14}$$

$$\therefore \sin^2 A = 1 - \frac{13^2}{14^2} = \frac{27}{14^2}$$

Sol.13 B

$$R = \frac{a}{2\sin A} = \frac{s-r}{2\sin A}$$

$$= \frac{s-r}{2\sin\frac{\pi}{2}} = \frac{s-r}{2}$$



Aliter : Let $\angle A = \frac{\pi}{2}$

$$r = (s-a)\tan\frac{A}{2} = (s-a)\tan\frac{\pi}{4} = s-a$$

$\Rightarrow$ a = s – r

$$\& \ R = \frac{abc}{4\Delta} = (s-r)\ \frac{bc}{4\Delta} = \frac{(s-r)}{2}$$

$$\{\because \Delta = \frac{1}{2} \text{ bc sin A} \Rightarrow 2\Delta = \text{bc sin } \frac{\pi}{2} \Rightarrow 2\Delta = bc\}$$

Sol.14 C

GivenΔ = 100 sq. cm, r_1 = 10 cm, r_2 = 50 cm

$$r_1 = \frac{\Delta}{s-a} \Rightarrow s-a = \frac{100}{10} \Rightarrow s-a = 10 \text{ ....(1)}$$

$$r_2 = \frac{\Delta}{s-b} \Rightarrow s-b = \frac{100}{50} \Rightarrow s-b = 2 \text{ ....(2)}$$

(1) - (2) $\Rightarrow$ b – a = 10 – 2 = 8

Sol.15 B

$$r = \frac{\Delta}{s},\ r_1 = \frac{\Delta}{s-a},\ r_2 = \frac{\Delta}{s-b},\ r_3 = \frac{\Delta}{s-c}$$

$$\Rightarrow r\ .\ r_1\ .\ r_2\ .\ r_3 = \frac{\Delta}{s}\ .\ \frac{\Delta}{(s-a)}\cdot\frac{\Delta}{(s-b)}\cdot\frac{\Delta}{(s-c)}$$

$$\Rightarrow r\ .\ r_1\ .\ r_2\ .\ r_3 = \Delta^2\ \{\Delta^2 = s(s-a)(s-b)(s-c)\}$$

Sol.16 A

$$\text{If } r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$$

$\Rightarrow$ (s – a) < (s – b) < (s – c)

$\Rightarrow$ –a < – b < –c $\Rightarrow$ a > b > c

Sol.17 B

$$A.M. = \frac{a+b+c}{3}$$

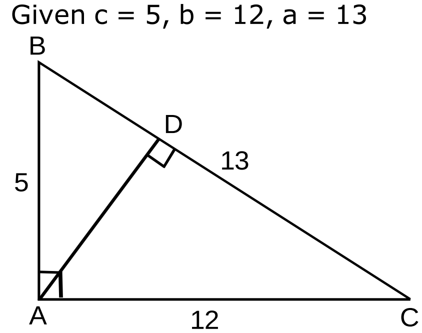
Altitudes are $\frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$

$$H.M. = \frac{3}{\frac{a}{2\Delta}+\frac{b}{2\Delta}+\frac{c}{2\Delta}} = \frac{6\Delta}{(a+b+c)}$$

$$(A.M.) \times (H.M.) = \frac{(a+b+c)}{3}\times\frac{3\times2\Delta}{(a+b+c)} = 2\Delta$$

Sol.18 B

Given c = 5, b = 12, a = 13



$$\cos A = \frac{12^2+5^2-13^2}{2(12)(5)} = 0$$

i.e. A = 90º

Altitude AD = $\frac{2\Delta}{a} = \frac{2\times 30}{13} = \frac{60}{13}$

$\{\Delta = \frac{1}{2} \times 5 \times 12 = 30\}$

Sol.19 C

$AD = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$\Rightarrow 4AD^2 = 2b^2 + 2c^2 - a^2$
$\Rightarrow 4BE^2 = 2c^2 + 2a^2 - b^2$
$\Rightarrow 4CF^2 = 2a^2 + 2b^2 - c^2$
$\therefore 4(AD^2 + BE^2 + CF^2) = 3(a^2 + b^2 + c^2)$

$\Rightarrow \frac{AD^2+DE^2+CF^2}{BC^2+CA^2+AB^2} = \frac{3}{4} = 3:4$

Sol.20 A

Given $\angle B = \frac{\pi}{2}$

$r = (s - b) \tan\frac{B}{2} = (s - b)\tan\frac{\pi}{4}$

$= \frac{a+b+c}{2} - b = \frac{c+a-b}{2} = \frac{AB+BC-CA}{2}$

Sol.21 A

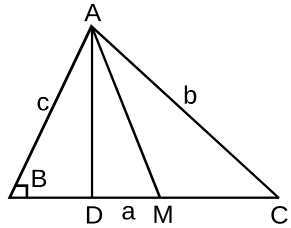
H is orthocentre of ΔABC
Radius of circumcircle of ABC,
BHC, CHA, AHB are same and equal to R
$\angle BHC = B + C$
In ΔBHC, by sin law

$\Rightarrow \frac{a}{\sin(B+C)} = 2R$ & $\frac{a}{\sin A} = 2R$

$\Rightarrow \sin(B + C) = \sin A \Rightarrow R_1 = R$
similarly $R_2 = R$ & $R_3 = R$

Sol.22 B

BM = a/2
BD = c cos B



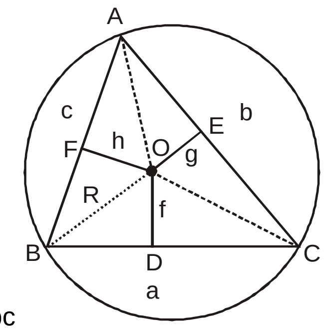
$DM = \frac{a}{2} - c\cos B$

$= \frac{a}{2} - c\,\frac{(c^2+a^2-b^2)}{2ca}$

$= \frac{a^2-c^2-a^2+b^2}{2a} = \frac{b^2-c^2}{2a}$

Sol.23 A

In ΔBOD,



$\tan B = \frac{\frac{a}{2}}{f} = \frac{a}{2f}$

$\Sigma\tan A = \pi\tan A$

$\Rightarrow \frac{a}{2f}+\frac{b}{2g}+\frac{c}{2h} = \frac{abc}{8fgh}$

$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \frac{abc}{4fgh}$

$\Rightarrow \frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda\frac{abc}{fgh} \quad \therefore \lambda = \frac{1}{4}$

Sol.24 A

$\frac{a-b}{b-c} = \frac{s-a}{s-c} \Rightarrow \frac{(s-b)-(s-a)}{(s-c)-(s-b)} = \frac{(s-a)}{(s-c)}$

$\Rightarrow (s - b)(s - c) - (s - a)(s - c)$
$= (s - a)(s - c) - (s - a)(s - b)$
$\Rightarrow (s - b)(s - c) + (s - a)(s - b) = 2(s - a)(s - c)$
divided by $(s - a)(s - b)(s - c)$

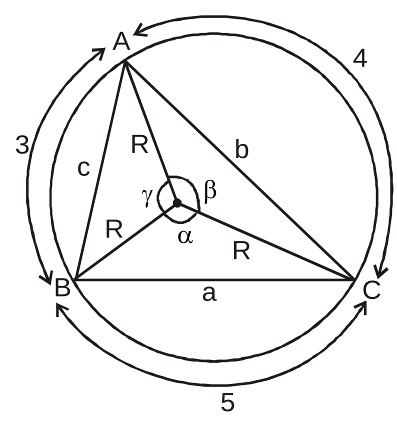
$\Rightarrow \frac{1}{(s-a)} + \frac{1}{(s-c)} = \frac{2}{(s-b)}$

$\Rightarrow \frac{\Delta}{(s-a)} + \frac{\Delta}{(s-c)} = \frac{2\Delta}{(s-b)}$

$\Rightarrow r_1 + r_3 = 2r_2 \Rightarrow r_1, r_2, r_3$ in A.P.

Sol.25 A

$2\pi R = 3 + 4 + 5 \Rightarrow R = \frac{6}{\pi}$



$\alpha = \frac{3\times 360}{3+4+5} = \frac{3\times 360}{12}$
$\alpha = 90, \beta = 120, \gamma = 150$
$\Delta ABC = \Delta OBC + \Delta OCA + \Delta OAB$

$$\Delta ABC = \frac{1}{2}R.R\sin\alpha + \frac{1}{2}R.R\sin\beta + \frac{1}{2}R.R\sin\gamma$$

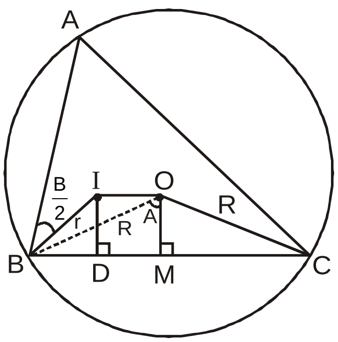
$$= \frac{1}{2}R^2[\sin 90º + \sin 120º + \sin 150º]$$

$$= \frac{R^2}{2}\left[1+\frac{\sqrt{3}}{2}+\frac{1}{2}\right] = \frac{1}{2}\left(\frac{6}{\pi}\right)^2\left[\frac{3+\sqrt{3}}{2}\right]$$

$$= \frac{36}{4\pi^2}(3+\sqrt{3}) = \frac{9\sqrt{3}}{\pi^2}(1+\sqrt{3})$$

Sol.26 B

In ΔOBM, $\cos A = \frac{r}{R}$



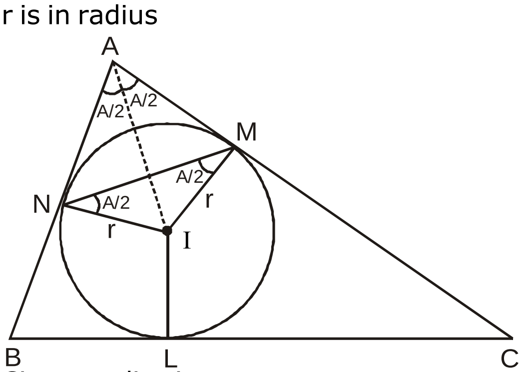
$$\Rightarrow \cos A + \cos B + \cos C = 1 + 4\pi\sin\frac{A}{2}$$

$$\Rightarrow \frac{r}{R} + \cos B + \cos C = 1 + \frac{r}{R}$$

$$\Rightarrow \cos B + \cos C = 1$$

Sol.27 C

r is in radius



Circumradius is x

$$x = \frac{r}{2\sin\frac{A}{2}} \quad |||'y,\ y = \frac{r}{2\sin\frac{B}{2}} \ \&\ z = \frac{r}{2\sin\frac{C}{2}}$$

$$\Rightarrow xyz = \frac{r^3}{8\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)} = \frac{r^3}{8\frac{r}{4R}} = \frac{1}{2}r^2R$$

Sol.28 B

$$r = \frac{\Delta}{s},\ r_1 = \frac{\Delta}{s-a},\ s = \frac{a+b+c}{2}$$

$$\frac{r}{r_1} = \frac{s-a}{s} = \frac{1}{2} \Rightarrow s = 2a$$

$$\because \tan\frac{A}{2} = \frac{r}{s-a}$$

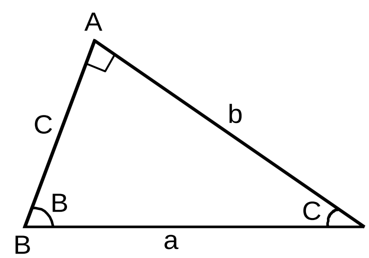
$$\tan\frac{A}{2}\left[\tan\frac{B}{2}+\tan\frac{C}{2}\right] = \frac{r}{s-a}\left[\frac{r}{s-b}+\frac{r}{s-c}\right]$$

$$= \frac{r^2(2s-b-c)}{(s-a)(s-b)(s-c)} = \frac{r^2(a+b+c-b-c)}{\left(\frac{\Delta^2}{s}\right)}$$

$$= \frac{\Delta^2}{s^2}\ \frac{s}{\Delta^2} \times a = \frac{a}{s} = \frac{a}{2a} = \frac{1}{2}$$

Sol.29 D

$$\Delta ABC,\ \angle A = \frac{\pi}{2}$$



$$\tan\frac{C}{2} = \frac{r}{s-c} = \frac{\Delta}{s(s-c)} \quad \{\because \Delta = \frac{1}{2}bc\}$$

$$\tan\frac{C}{2} = \frac{\frac{1}{2}bc}{\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}} = \frac{2bc}{2b^2+2ab}$$

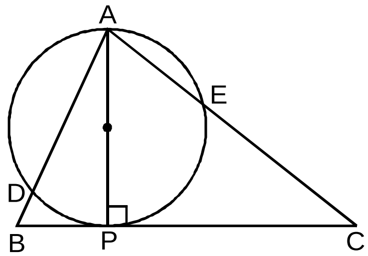
$$\{\because a^2 = b^2 + c^2\}$$

$$= \frac{2bc}{2b(b+a)} = \frac{c}{a+b} \times \frac{a-b}{a-b}$$

$$= \frac{c(a-b)}{a^2-b^2} = \frac{c(a-b)}{c^2} = \frac{a-b}{c}$$

Sol.30 D

Sine Rule in ΔADE

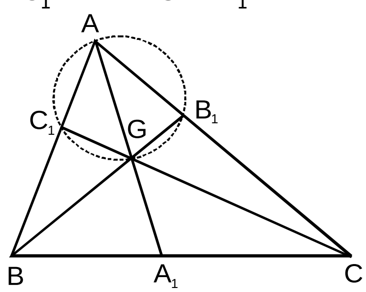


$$\frac{DE}{\sin A} = AP$$

$$\Rightarrow\ DE = \frac{2\Delta}{a}\ \sin A\ = 2\Delta \times \frac{1}{R} = \frac{\Delta}{R} \qquad \left\{\frac{\sin A}{a} = \frac{1}{2R}\right.$$

Sol.31 C

$BC_1 . BA = BG . BB_1$



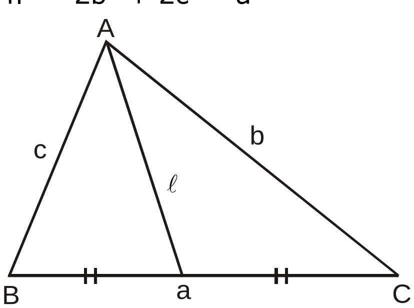
$$\frac{c}{2}\ .\ c = \frac{2}{3}\ \ell_B\ .\ell_B\ \Rightarrow 3c^2 = 4\ell_B{}^2$$

$$\Rightarrow 3c^2 = 4.\ \frac{1}{4}\ (2a^2 + 2c^2 - b^2)$$

$$\Rightarrow\ c^2 + b^2 = 2a^2$$

Sol.32 B

$4l^2 = 2b^2 + 2c^2 - a^2$



$$\Rightarrow\ 4\ell^2 = 2\ (b^2 + c^2 - a^2) + a^2$$

$$\Rightarrow\ 4\ell^2 = a^2 + 4bc \cos A$$

Sol.33 C

$a = 1$

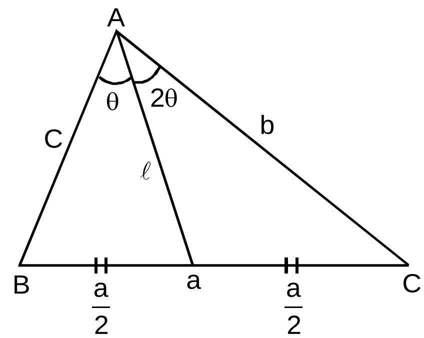
$$(a + b + c) = 6\left(\frac{\sin A + \sin B + \sin C}{3}\right)$$

$$\Rightarrow\ (a + b + c) = \frac{2}{2R}\ (a + b + c) \Rightarrow R = 1$$

$$\because\ \frac{a}{\sin A} = 2A\ \Rightarrow\ \sin A = \frac{1}{2}\ \Rightarrow A = \frac{\pi}{6}$$

Sol.34 B

$$\frac{\ell}{\sin B} = \frac{\frac{a}{2}}{\sin\theta}\ \&\ \frac{\ell}{\sin C} = \frac{\frac{a}{2}}{\sin 2\theta}$$

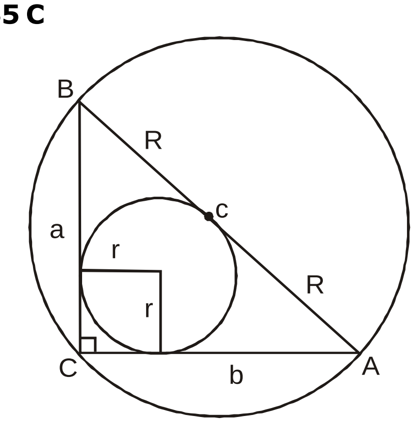


$$\frac{\sin B}{\sin C} = \frac{\sin\theta}{\sin 2\theta} \Rightarrow \qquad \frac{\sin B}{\sin C} = \frac{\sin\theta}{2\sin\theta\cos\theta}$$

$$\Rightarrow\ \frac{\sin B}{\sin C} = \frac{1}{2\cos\theta}\ \Rightarrow\ \frac{\sin B}{\sin C} = \frac{1}{2\cos\theta}$$

$$\Rightarrow\ \frac{\sin B}{\sin C} = \frac{1}{2}\ \sec\ \frac{A}{3}$$

Sol.35 C



$$r = (s - c)\ \tan\ \frac{C}{2}\ \ \Rightarrow r = s - c$$

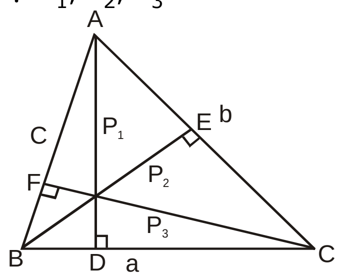
$$\Rightarrow 2r = a + b + c - 2c = a + b - c = a + b - 2R$$

$$\Rightarrow\ \ 2(r + R) = a + b$$

Sol.36 C

AD = P_1, BE = P_2, CF = P_3

$\because$ P_1, P_2, P_3 in H.P.

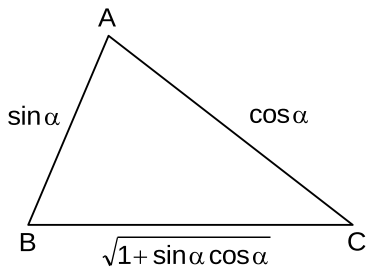


$\Rightarrow \frac{1}{P_1}, \frac{1}{P_2}, \frac{1}{P_3}$ in A.P. $\Rightarrow \frac{a}{2\Delta}, \frac{b}{2\Delta}, \frac{c}{2\Delta}$ in A.P.

$\Rightarrow$ a, b, c in A.P. $\Rightarrow$ sinA, sinB, sinC in A.P.

Sol.37 C

Greatest side is BC > 1



$0 < \alpha < \frac{\pi}{2}$

$$\cos A = \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

$\cos A = -\frac{1}{2} \quad \Rightarrow \quad A = 120°$

Sol.38 B

$$r = \frac{a}{2}\cot\frac{\pi}{n},\ R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

$$r + R = \frac{a}{2}\left(\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right)$$

$$= \frac{a}{2}\left(\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right) = \frac{a}{2}\cot\frac{\pi}{2n}$$

Sol.39 A

$$a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$$

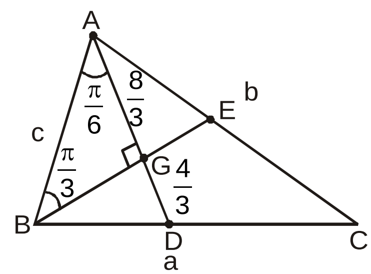
$$\Rightarrow a\,\frac{s(s-c)}{ab} + c\,\frac{s(s-a)}{bc} = \frac{3b}{2}$$

$\Rightarrow$ 2s [2s − c − a] = $3b^2$

$\Rightarrow$ (a + b + c) = 3b

$\Rightarrow$ a + c = 2b $\Rightarrow$ a, b, c in A.P.

Sol.40 C



AD = 4

In ΔABG, $\frac{\frac{8}{3}}{c} = \sin\frac{\pi}{3}$

$\Rightarrow C = \frac{8}{3}\,\frac{2}{\sqrt{3}} = \frac{16}{3\sqrt{3}}$

Area ΔABD = Area ΔADC

$\Delta ABC = 2\Delta ABD = 2 \times \frac{1}{2}$ (AD) (AB) $\sin\frac{\pi}{6}$

$= 4 \cdot \frac{16}{3\sqrt{3}} \cdot \frac{1}{2} = \frac{32}{3\sqrt{3}}$ sq. units

Sol.41 A

PQ = PR = R

ΔOPQ & ΔOPR are Equilateral triangle

$\angle P = \angle QPO + \angle OPR = 60° + 60° = 120°$

Sol.42 B

$r = \sqrt{3}$, ID = IE = IF & $r = \frac{\Delta}{s}$

$\Rightarrow \Delta = rs = \sqrt{3}\,s$

In Δ IBD, $\tan 15^\circ = \dfrac{\sqrt{3}}{\left(b\dfrac{\sqrt{3}}{2}\right)}$

$\Rightarrow b = \dfrac{2}{\tan 15^\circ} = 2(2 + \sqrt{3})$

$\therefore s = \dfrac{2b + b\sqrt{3}}{2} = \dfrac{1}{2}(b(2 + \sqrt{3}))$

$= \dfrac{2}{2}(2 + \sqrt{3})^2 = (7 + 4\sqrt{3})$

$\Delta = \sqrt{3}\,(7 + 4\sqrt{3}) = 12 + 7\sqrt{3}$ sq. units

Sol.43 C

$x^3 - 11x^2 + 38x - 40 = 0 \begin{cases} a \\ b \\ c \end{cases}$

$\Sigma a = 11,\ \Sigma ab = 38,\ abc = 40$

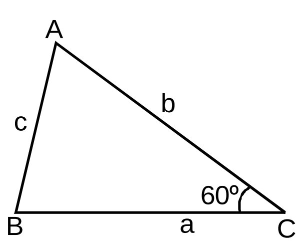
$\Sigma \dfrac{\cos A}{a} = \dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c}$

$= \dfrac{\dfrac{1}{2}[b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2]}{abc}$

$= \dfrac{11^2 - 2(38)}{2\,.\,(40)} = \dfrac{45}{2 \times 40} = \dfrac{9}{16}$

Sol.44 A

$\dfrac{a\sin B + b\sin A}{\sqrt{\sin A \sin B}} = 4$



$\Rightarrow \dfrac{\dfrac{ab}{2R} + \dfrac{ba}{2R}}{\sqrt{\dfrac{ab}{4R^2}}} = 4$

$\Rightarrow \dfrac{2ab}{2R}\,.\,\dfrac{2R}{\sqrt{ab}} = 4$

$\Rightarrow \sqrt{ab} = 2$

$a^2 + b^2 + c^2 = 2ab\cos 60^\circ = 2\,.\,4.\,\dfrac{1}{2} = 4$

Sol.45 A

cotA : cot B : cot C = 30 : 19 : 6

$\begin{cases} \cot A = 30k \\ \cot B = 19k \\ \cot C = 6k \end{cases}$

$\because \Sigma \cot A \cot B = 1$

$\Rightarrow 570\,k^2 + 114\,k^2 + 180k^2 = 1$

$\Rightarrow k^2 = \dfrac{1}{864} \Rightarrow k = \dfrac{1}{12\sqrt{6}}$

$\cot A = \dfrac{30}{12\sqrt{6}} \Rightarrow \sin A = \dfrac{2\sqrt{6}}{7} = \dfrac{10\sqrt{6}}{35}$

$\cot B = \dfrac{19}{12\sqrt{6}} \Rightarrow \sin B = \dfrac{12\sqrt{6}}{35} = \dfrac{12\sqrt{6}}{35}$

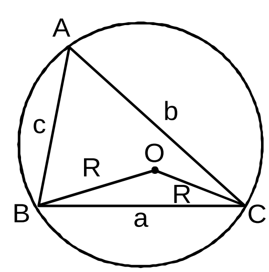
$\cot C = \dfrac{6}{12\sqrt{6}} \Rightarrow \sin B = \dfrac{2\sqrt{6}}{5} \Rightarrow \dfrac{14\sqrt{6}}{35}$

a : b : c = sinA : sin B : sin C

= 10 : 12 : 14 = 5 : 6 : 7

Sol.46 A

$2(2R)^2 = a^2 + b^2 + c^2$



$\Rightarrow \left(\dfrac{a}{2R}\right)^2 + \left(\dfrac{b}{2R}\right)^2 + \left(\dfrac{c}{2R}\right)^2 = 2$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

Sol.47 C

$$\frac{a}{1} = \frac{b}{\sqrt{3}} = \frac{c}{2} = k$$

$$\cos A = \frac{3+4-1}{4\sqrt{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{6}$$

$$\cos B = \frac{1+4-3}{2.1.2} = \frac{1}{2} \Rightarrow B = \frac{\pi}{3} \therefore c = \frac{\pi}{2}$$

$\Rightarrow$ A, B, C in A.P.

Sol.48 A

(s – a), (s – b), (s – c) in G.P.

$(s-b)^2 = (s-a)(s-c)$

$$\left(\frac{c+a-b}{2}\right)^2 = \left(\frac{b+c-a}{2}\right)\left(\frac{a+b-c}{2}\right)$$

$\Rightarrow a^2 + b^2 + c^2 + 2ac - 2ab - 2bc = b^2 - (a-c)^2$

$\Rightarrow a^2 + b^2 + c^2 + 2ac - 2ab - 2bc$

$= b^2 - a^2 - c^2 + 2ac$

$$\Rightarrow \frac{a^2+c^2}{a+c} = b \Rightarrow \frac{(2R)^2[\sin^2 A + \sin^2 C]}{(2R)[\sin A + \sin C]} = b$$

$$\Rightarrow \frac{\sin^2 A + \sin^2 C}{\sin A + \sin C} = \frac{b}{2R} = \sin B$$

Aliter

$$\frac{s-b}{s-a} = \frac{s-c}{s-b} \quad \text{Apply C \& D}$$

$$\Rightarrow \frac{2s-a-b}{a-b} = \frac{2s-b-c}{b-c} \Rightarrow \frac{c}{a-b} = \frac{a}{b-c}$$

$\Rightarrow bc - c^2 = a^2 - ab$

$\Rightarrow a^2 + c^2 = b(a + c)$

$\Rightarrow a^2 + c^2 = b(a + c)$

$\Rightarrow (2R)^2 (\sin^2 A + \sin^2 C) = (2R)^2 \sin B (\sin A + \sin C)$

$$\Rightarrow \frac{\sin^2 A + \sin^2 C}{\sin A + \sin C} = \sin B$$

Sol.49 B

$$\cos A = \frac{\sin B}{2\sin C} \Rightarrow 2\sin C \cos A = \sin B$$

$$\Rightarrow 2\frac{c}{2R}\,\frac{b^2+c^2-a^2}{2bc} = \frac{b}{2R}$$

$\Rightarrow b^2 + c^2 - a^2 = b^2$

$\Rightarrow c = a \Rightarrow \Delta ABC$ is isosceles

Sol.50 B

$$\frac{\cos A}{a} = \frac{\tan C}{c} \Rightarrow \frac{\cos A}{2R\sin A} = \frac{\tan C}{2R\sin C}$$

$\Rightarrow \sin A = \cos A \cos C$

$\Rightarrow \sin(B + C) = \cos A \cos C$

Sol.51 D

$$1 - \tan\frac{A}{2}\tan\frac{B}{2} = 1 - \frac{\Delta}{s(s-a)}\,\frac{\Delta}{s(s-b)}$$

$$= 1 - \frac{\Delta^2(s-c)}{s(s-a)(s-b)(s-c)s}$$

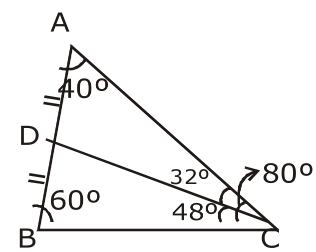
$$= 1 - \frac{s-c}{s} = \frac{s-s+c}{s} = \frac{c}{s} = \frac{2c}{a+b+c}$$

Sol.52 C

$A + B + C = 180^\circ$ A, B, C in A.P.

$2B = A + C$ & $C = 2A$

$2B = 3A$



$$A + \frac{3A}{2} + 2A = 180^\circ \Rightarrow \frac{9A}{2} = 180^\circ$$

$\Rightarrow A = 40^\circ \therefore B = 60^\circ, C = 80^\circ$

$CD = 2\sqrt{3}$ cm

$$\frac{\angle BCD}{\angle DCA} = \frac{2}{3} \Rightarrow \angle BCD = 32^\circ; \angle DCA = 48^\circ$$

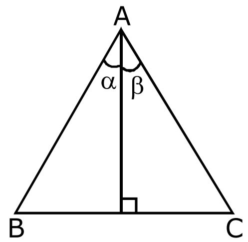
In ΔBCD (using sine rule)

$$\frac{2\sqrt{3}}{\sin 60^\circ} = \frac{\frac{c}{2}}{\sin 32^\circ}$$

$$\Rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{c}{2\sin 32^\circ}$$

$\Rightarrow c = 8\sin 32^\circ$

Sol.53 B



$(\alpha - \beta) = \dfrac{\pi}{6} \Rightarrow \alpha = \beta + \dfrac{\pi}{6}$

$\tan\alpha = 3\tan\beta \Rightarrow \tan\left(\beta + \dfrac{\pi}{6}\right) = 3\tan\beta$

$\Rightarrow \dfrac{\tan\beta + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}\tan\beta} = 3\tan\beta$

$\Rightarrow \sqrt{3}\tan\beta + 1 = 3\tan\beta\,(\sqrt{3} - \tan\beta)$

$\Rightarrow 3\tan^2\beta - 2\sqrt{3}\tan\beta + 1 = 0$

$\Rightarrow (\sqrt{3}\tan\beta - 1)^2 = 0$

$\Rightarrow \tan\beta = \dfrac{1}{\sqrt{3}} \Rightarrow \beta = 30^\circ, \alpha = 60^\circ$

ΔABC is right angled Δ

Sol.54 C

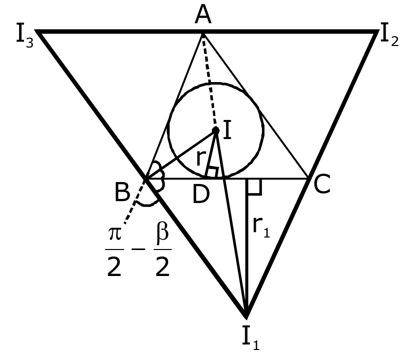
$$\frac{2\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right)\left(1 - \tan\frac{A}{2}\tan\frac{B}{2}\right)}{\left(1 + \tan^2\frac{A}{2}\right)\left(1 + \tan^2\frac{B}{2}\right)}$$

$$= 2\tan\left(\frac{A+B}{2}\right)\left(1 - \tan\frac{A}{2}\tan\frac{B}{2}\right)^2 \cos^2\frac{A}{2}\cos^2\frac{B}{2}$$

$$= 2\frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}\left(\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}\right)^2$$

$$= 2\frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}\cos^2\left(\frac{A+B}{2}\right) = 2\cos\frac{C}{2}\sin\frac{C}{2} = \sin C$$

Sol.55 B



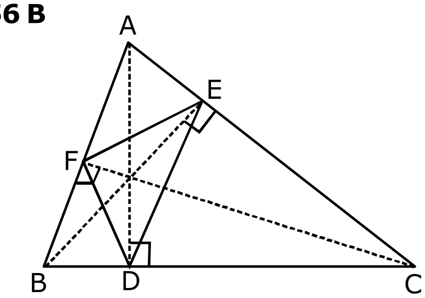
$\dfrac{r}{BI} = \sin\dfrac{B}{2} \Rightarrow BI = r\,\mathrm{cosec}\,\dfrac{B}{2}$

& $\sin\left(\dfrac{\pi}{2} - \dfrac{\beta}{2}\right) = \dfrac{r_1}{BI_1} \Rightarrow BI_1 = \dfrac{r_1}{\cos\frac{B}{2}}$

$II_1 \,.\, II_2 \,.\, II_3 = (4R)^3 \sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

$= 16R^2 r$

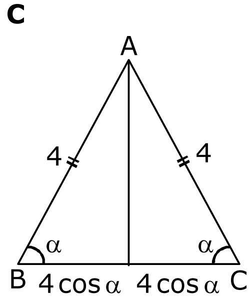
Sol.56 B



$a' = R\sin 2A,\ b' = R\sin 2B$
$c' = R\sin 2C,\ B' = \pi - 2B$

$\Delta = \dfrac{1}{2}R^2 \sin 2A \sin 2C \,.\, \sin 2B$

Sol.57 C



$Rr = \dfrac{abc}{4\Delta} \times \dfrac{\Delta}{s} = \dfrac{abc}{2(a+b+c)}$

$= \dfrac{4.4.8\cos\alpha}{2(8 + 8\cos\alpha)} = \dfrac{8.16\cos\alpha}{16(1+\cos\alpha)} = \dfrac{8\cos\alpha}{1+\cos\alpha}$